10.8 POLAR GRAPHS

NEED 16 POINTS IF PLOTTING POINTS ONLY

$r = \sin\theta$

FOR GRAPHING

$\sqrt{2} \approx 1.4$

$\sqrt{3} \approx 1.8$

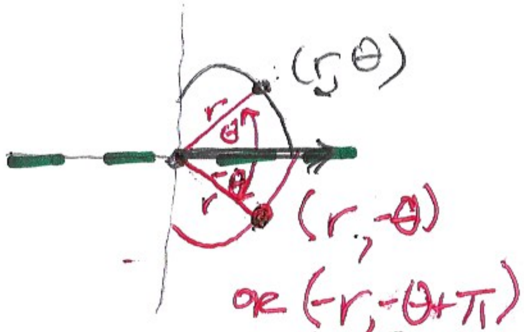
θ	$r = \sin\theta$	(r, θ)
0	$\sin 0 = 0$	$(0, 0)$
$\frac{\pi}{6}$	$\sin\frac{\pi}{6} = \frac{1}{2}$	$\left(\frac{1}{2}, \frac{\pi}{6}\right) = \left(0.5, \frac{\pi}{6}\right)$
$\frac{\pi}{4}$	$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$	$\left(\frac{\sqrt{2}}{2}, \frac{\pi}{4}\right) \approx \left(0.7, \frac{\pi}{4}\right)$
$\frac{\pi}{3}$	$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$	$\left(\frac{\sqrt{3}}{2}, \frac{\pi}{3}\right) \approx \left(0.9, \frac{\pi}{3}\right)$
$\frac{\pi}{2}$	$\sin\frac{\pi}{2} = 1$	$\left(1, \frac{\pi}{2}\right)$
$\frac{2\pi}{3}$		
$\frac{3\pi}{4}$		
$\frac{5\pi}{6}$		
π		
$\frac{7\pi}{6}$		
$\frac{5\pi}{4}$ ...		

ALL IN Q_1

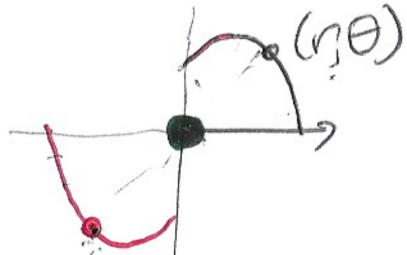
IN Q_2

IN Q_3

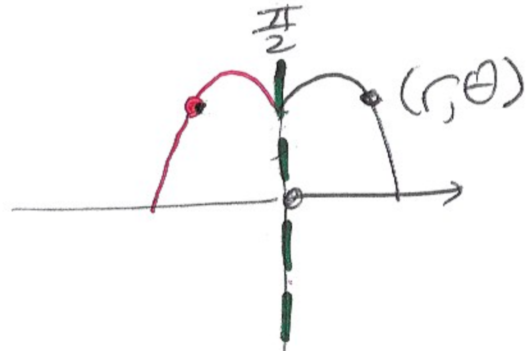
3 TYPES OF SYMMETRY IN POLAR:



OVER POLAR AXIS



OVER POLE



OVER $\theta = \frac{\pi}{2}$

SYM TEST

REPLACE (r,θ)
WITH $(r,-\theta)$, $(-r,\pi-\theta)$
AND IF THE EQUATION
SIMPLIFIES BACK TO
THE ORIGINAL EQ'N,
THEN THE GRAPH
IS SYM OVER POLAR
AXIS

TEST $r=\sin\theta$ FOR SYM OVER POLAR AXIS

$(r,-\theta)$: $r=\sin(-\theta)$
$r=-\sin\theta$

FROM TRIG:
$\sin(-\theta)=-\sin\theta$

↑ DOESN'T SIMPLIFY BACK TO ORIGINAL $r=\sin\theta$
NO CONCLUSION
(CAN'T SAY GRAPH IS OR IS NOT SYM BASED ON THIS ALONE)

CONT'D

$(-r, \pi-\theta)$: $-r = \sin(\pi-\theta)$

$-r = \cancel{\sin\pi}^{0} \cos\theta - \cancel{\cos\pi}^{-1} \sin\theta$

$-r = \sin\theta$

$r = -\sin\theta$

↳ DOESN'T SIMPLIFY BACK TO $r=\sin\theta$

NO CONCLUSION

FROM TRIG:

$\sin(A \pm B)$

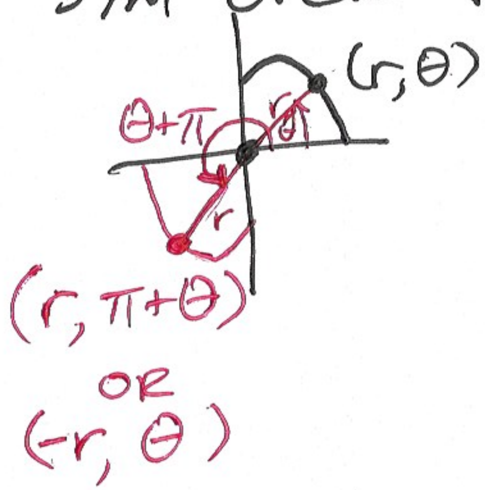
$= \sin A \cos B \pm \cos A \sin B$

☆ SO, NO CONCLUSION ABOUT SYM OVER POLAR AXIS

(BECAUSE THERE ARE ACTUALLY AN INFINITE NUMBER OF SYM TESTS FOR SYM OVER POLAR AXIS)

NOTE: IF EITHER TEST HAD INDICATED THE GRAPH IS SYM OVER POLAR AXIS, SKIP THE OTHER TEST

SYM OVER POLE



SYM TEST:

REPLACE (r, θ) WITH $(r, \pi + \theta)$ $(-r, \theta)$ AND IF THE EQUATION SIMPLIFIES BACK TO THE ORIGINAL EQUATION, THEN THE GRAPH IS SYM OVER POLE.

TEST $r = \sin\theta$ FOR SYM OVER POLE

$(-r, \theta)$: $-r = \sin\theta$

$r = -\sin\theta \longrightarrow$ NO CONCLUSION

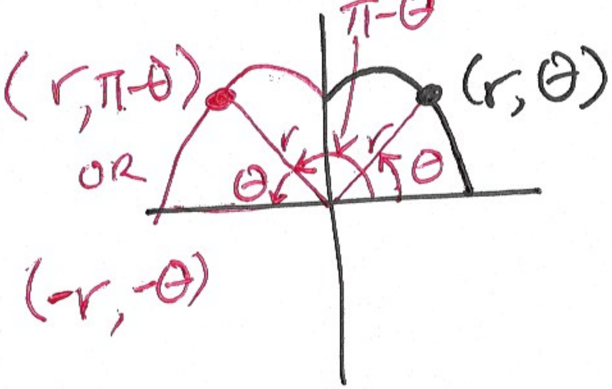
$(r, \pi + \theta)$: $r = \sin(\pi + \theta)$

$r = \underbrace{\cancel{\sin\pi}}_{0} \cos\theta + \underbrace{\cancel{\cos\pi}}_{-1} \sin\theta$

$r = -\sin\theta \longrightarrow$ NO CONCLUSION

SO, NO CONCLUSION ABOUT SYM OVER POLE

SYM OVER $\theta = \frac{\pi}{2}$



SYM TEST:

REPLACE (r, θ) WITH $(r, \pi - \theta)$ $(-r, -\theta)$ AND IF THE EQUATION SIMPLIFIES BACK TO THE ORIGINAL EQUATION, THEN THE GRAPH IS SYM OVER $\theta = \frac{\pi}{2}$

TEST $r = \sin\theta$ FOR SYM OVER $\theta = \frac{\pi}{2}$

$-r = \sin(-\theta)$

$-r = -\sin\theta$

$r = \sin\theta \longrightarrow$ SAME AS ORIGINAL EQUATION

GRAPH IS SYM OVER $\theta = \frac{\pi}{2}$

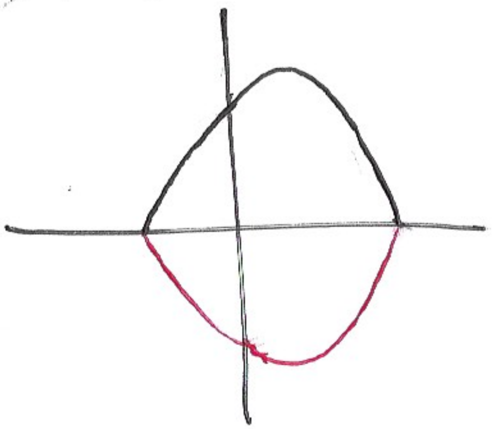
$\downarrow$

NO NEED TO RUN $(r, \pi - \theta)$ TEST

SYM OVER	REPLACE (r,θ) WITH	
POLAR AXIS	$(r,-\theta)$	$(-r,\pi-\theta)$
POLE	$(-r,\theta)$	$(r,\pi+\theta)$
$\theta=\frac{\pi}{2}$	$(-r,-\theta)$	$(r,\pi-\theta)$

& TRY TO SIMPLIFY BACK TO ORIGINAL EQUATION

IF GRAPH IS SYM OVER POLAR AXIS AND WE KNOW WHAT THE GRAPH LOOKS LIKE IN Q_1, Q_2 THEN WE CAN REFLECT IT INTO Q_4, Q_3



SO WE ONLY NEED TO PLOT POINTS FOR θ IN Q_1, Q_2

IE. $\theta \in [0,\pi]$

BEFORE USING REFLECTIONS

MINIMUM INTERVAL FOR PLOTTING POINTS